AMERICAN SOCIETY OF CIVIL ENGINEERS.

INSTITUTED 1852.

TRANSACTIONS.

NOTE.—This Society is not responsible, as a body, for the facts and opinions advanced in any of its publications.

CC.

(Vol. IX.—August, 1880.)

DISCUSSION

ON

PAPER CLXXXVI.—THE USE OF STEEL FOR BRIDGES.*

By WILLIAM SELLERS, Member of the Society.

The paper which Mr. Cooper has contributed upon the use of Steel for Bridges, is so comprehensive and so justly conservative, that little remains but to emphasize his conclusions.

The position he takes as to the province of the engineer in determining the quality of the material he is to deal with, is eminently sound, and that it is so little observed is to be attributed, perhaps, to vicious, but long-established custom, for there are to-day very few specifications, whether for iron or steel structures, but in some way prescribe a process

* Paper CLXXXVI.—The Use of Steel for Bridges by Theodore Cooper, Member A. S. C. E., Vol. VIII., Page 263, October, 1879.

of manufacture; it is gratifying, therefore, to find an engineer who has had such large experience as Mr. Cooper in testing these metals, taking ground against a practice that stands in the way of all improvement in the process of manufacture. The engineer should deal only with the material as he finds it, and subject it to such tests as will determine its physical characteristics or such characteristics as qualify it for the structure in which he intends to use it; it becomes, therefore, a matter of the greatest moment to him what these tests shall be. In dealing with this portion of the subject, Mr. Cooper has omitted one important element, that is, capacity to bear fatigue, or perhaps he assumes that the ductility, as compared with the ultimate strength, will measure it, and it may be that this is so; but, with our present amount of knowledge upon this subject, I do not think that we can confidently affirm it; the proportion of ductility to ultimate strength is our most important factor, because of this we have the largest number of recorded experiments for comparison, but the capacity to withstand shocks or blows does not seem to bear any direct relation to this proportion.

There is little doubt but that steels of the higher carbonization withstand better than those of low, the kind of shock to which piston rods and crank pins are subjected, but this is not the kind of shock to which the steel in bridge structures is subjected in use, or that may be thrown upon it by accident; the safety of the one is assured by its capacity to withstand heavy strains without bending, that of the other by its capacity to withstand heavy blows under which it must bend without breaking; the capacity, therefore, to withstand such blows is one of the characteristics which the engineer's tests should determine; and the drop test with bending blows only, and reversing the piece after each blow until broken, should not be omitted until at least, a sufficient number of tests have been recorded to establish the fact, if it be a fact, that the capacity to bear fatigue has a definite relation to the proportion which the ductility bears to the ultimate strength.

The very sufficient reasons that the author has given for desiring the tests for ductility made upon the same form and size of specimens are equally applicable to the drop bending tests.

They should be made upon the same size of specimen with the same distance between the points of support, and with the same weight falling from the same height; perhaps no one is better qualified than the author to determine what these details should be, indeed it is quite probable

that this very kind of test is the one to which he alludes, when he speaks of the desirability of testing all steels by impact or falling weight, but from the context it is uncertain whether he proposes to test to destruction by an extremely heavy blow or by a number of blows, each heavy enough to produce a decided bend; for the purpose of comparison, however, it is important that the character of this test should be defined in all its details. I quite agree with Mr. Cooper in affirming "the import-"ance and necessity of requiring a ductile metal regardless of what its "tensile strength may be," but I do not think sufficient importance has been given to the proportion of ductility to ultimate strength; this proportion will be largely affected by the chemical composition of the material; that is to say, with an ultimate strength of 80 000 lbs., the ductility will vary from ten to thirty per cent., depending upon the chemical composition of the material, so that with a *given amount of ductility* the safest material for the engineer to use will almost certainly be that having the highest ultimate strength, the requirements therefore which the author has formulated for bridge steel, whilst safe, are misleading, and if expressed, as he himself has stated it in the sentence I have quoted, would have a tendency to improve the quality of the manufacture, thus, for plates, angles, channels and other shapes, an elongation of not less than twenty per cent. in eight inches, an ultimate strength of not less than 65 000 lbs., and a limit of elasticity not less than half the ultimate strengh.

The reference to the form of thread that should be used for bridge screws, is of questionable value. The truncated V form which the author condemns, is the form which our naval engineers have used for nearly 15 years, and has been almost universally adopted in the United States; the section of the metal remaining, is less reduced than with the form he recommends, whilst the angles are very obtuse; it should require conclusive experiments establishing the authors opinion to warrant a change in this particular, where uniformity has been almost attained in the direction excepted to, and it is very certain immense numbers of steel screws have been made with this form of thread without failure.

I could have wished that the author had elaborated his remarks concerning the annealing of steel as it might be inferred that all eye bars, for example, would require to be annealed, because unequally or locally heated; if locally strained from any cause, steel should be annealed, but it must be borne in mind that annealing will reduce the ultimate strength

and the limit of elasticity giving probably a corresponding increase of ductility, so that in these particulars the metal annealed will not correspond with other similar materials in other parts of the structure that are not annealed.

The density of steel is largely affected by the kind and amount of treatment the ingot receives in fashioning it to the finished shape, in this process the particles are driven closer together and the application of heat without concurrent working, will drive them apart. When, therefore, annealing is resorted to, it should be done at a very low heat and for as short a time as possible; too high a heat or too long contiuued, or too frequent, will produce a marked degradation of the material; indeed the injury that may result from the practice, except in the hands of an expert, is so great that it should not be tolerated in general practice. There are conditions in which annealing is indispensable to attain the best results, but these occur rarely in worked material, and with all, the ordinary workman is incompetent to deal.

AMERICAN SOCIETY OF CIVIL ENGINEERS.

INSTITUTED 1852.

TRANSACTIONS.

NOTE.—This Society is not responsible, as a body, for the facts and opinions advanced in any of its publications.

CCI.

(Vol. IX.—August, 1880.)

REMARKS ON THE CAUSES OF FALL OF THE WESTERN ARCHED APPROACH TO THE SOUTH STREET BRIDGE, PHILADELPHIA.*

By J. G. BARNARD, Hon. Mem. A. S. C. E.

The origin and circumstances of settlement of the pier No. 2, as described by Mr. Stauffer, "Transactions," September, 1878, to which is due the catastrophe, are so peculiar as to merit discussion.

The soundings which were made *subsequently* to the fall showed rock at from 36 to 40 feet below surface of the marsh over which the approach was carried. A depression in the rock surface extending from under about the middle of the northern half of the pier and beyond it northerly was filled to a depth of about 3 feet with "soft mud" mixed with gravel; next above was a stratum of very hard gravel and tough clay 7 feet thick over the mud socket, but only 3 feet thick towards the south end of the pier, where it was in immediate contact with the underlying rock. Thence, upwards, was mud, quite soft at surface, "but passing into a tough clay sometime before it reached the gravel."

* Paper CLXXI, D. McN. STAUFFER, Vol. VII., p. 264, September, 1878.

The pier rested on 84 piles driven—not to a prescribed final subsidence, but, until after repeated trials, it was *found impossible to force the pile any further*, under the blow of a 2 000-pound hammer, falling 32 feet.

A pile thus driven *ought* to exhibit "impossibility" of being forced further under almost any amount of permanent load, whereas the actual permanent load under which the pier settled was but 24 tons per pile.

Piles driven by me under the piers of the Forts Hamilton and Wadsworth wharves, with a 2 200-lb hammer and 20 feet fall, to a final subsidence of 2 inches were *calculated* to have an ultimate resistence of over 100 tons per pile, and were *intended* to bear a permanent load of 20 tons—an occasional one (yet one probably of considerable duration) of 33½ tons. The new London Bridge is said to press upon each pile of its pier foundation with a weight of 80 tons.

The attributing the yielding to loss of lateral friction by the intrusion of water as a result of "tremor in the piles caused by the heavy and constant travel over the approach," seems to me fanciful—by no means a legitimate deduction from any experimental results, and a sort of hypothesis an engineer should cautiously beware of.

There was a "tremor" indeed when a 2 000-pound hammer fell 32 feet on the heads of these piles. If *that* did not let in all the water that could *thus* be admitted, nothing could. The theory of "vibration" is carried to an extreme indeed when it is supposed that "heavy and constant travel" of carriages and wagons could extend a sensible (and water-admitting) vibration through 2 000 tons of masonry to these piles. Moreover, if the shock of such a blow did not "punch" through the crust of gravel between the bottom of the pile and the mud stratum beneath, no additional dead weight would do it. *That* suggestion is, therefore, quite untenable. Chicago buildings, on piles, have furnished just such experience. Moreover, the hypothesis of "lubrication" is something new and unsupported by any knowledge we have in connection with weight-bearing capacity of piles.

Omitting, however, his conjectural lubrication and its equally conjectural effects, and discarding the alternative of "punching" through the gravel, Mr. Stauffer is doubtless otherwise quite right in supposing that the hard crust itself settled into the soft mud beneath. The hard crust rested on the mud—a material as incompressible as water, but like water having, if confined, no other measure for its load-bearing power than the resistances of the sides of the confining envelopes. In

the case before us the envelope is the gravel stratum above the rock underneath. Gradually, doubtless, this semi-fluid was forced along between the rock and the lateral overlying strata of gravel and clay, thus allowing the superstructure to sink *pari passu* with the displacement of the mud.

It is, however, alleged, or assumed, that the structure remained quite stable for a period of "six or seven years." The pile driving was finished in September, 1870. Mr. Stauffer left after completion of the work in 1876. We are not informed what time the superstructure over these piles was completed and the full loads brought to bear on the foundations; but the bridge was completed in the latter part of 1875, and opened to travel early in 1876.

It is stated that the pier is reported "to have first begun to show signs of failing in the early part of 1877;" during the twelve months following (*i.e.*, to February, 1878,) the north end had settled 20 inches, *and had been pushed about six inches to the south*, "as if the platform and foundation and pier masonry were revolving about a point located in the south face of the approach, above the springing line of the arch."

Another peculiarity of the case is suggested by the foregoing. While the ends of the piles are so firmly held by hard gravel as to resist "re-"peated trials" with a 2 000-pound hammer falling 32 feet, the material which, above the gravel, surrounded the piles was soft mud, which could afford little lateral support to the piles; though while everything was in its normal condition, quite sufficient.

With one end of the pier firmly supported—the other sinking—the point of rotation of the foundation and pier, considered as a rigid mass, would naturally be the firmly (as to *vertical* motion) held south end. But this would involve a very considerable lateral (northward) motion of the continuous bridge floor above, comprising in that term the arches, spandrels, and roadway. The rigidity of the wide and continuous mass was too great to permit such motion, either by bending or rupture, whereas, the want of lateral resistance in the mud which surrounded the piles allowed lateral (southerly) motion in the foundation mass itself, accompanied by a slight *canting* of the piles.

Hence the fulcrum, or centre of motion, transferred itself from the foot of the pier-foundation to a point above—*i.e.*, as Mr. Stauffer describes it, "a point located in the south face of the approach, above the springing line of the arch."

The piles, once thrown, ever so little, out of the vertical, the weight of the superstructure exerts a component force, progressively and constantly augmenting to increase the displacement.

The fall of the structure finally ensued—not, it would seem, as a direct effect of vertical settlement merely, but from the angular displacement of the piles, reaching a limit where they could no longer offer, even temporarily, resistance to the enormous lateral thrust exerted on them. The pier finally sank down (moving southward) by the *overthrow* of the piles, as the location and condition of the parts after the fall clearly indicate.

The above view of the case differs only from Mr. Stauffer's in dissenting from his opinion (as I must do unless some record of instrumental observation exhibiting the *proof* be exhibited) as to the "stability of the structure for a period of years before falling"—unless by "stability" is only meant that the bridge did *stand* one, or two, or three years from the time the load on Pier No. 2 was imposed, without noticeable (or noticed) defect of stability.

The *original* pressure on the mud of the underlying "pocket" (*i. e.*, the weight of the overlying strata) was about 2 tons. The burden of the pier—2 000 tons—was *mainly* transmitted to the gravel stratum and thence for the northern portion to the mud. The area of gravel stratum enclosing the lower end of all the piles did not exceed 600 square feet. Hence the semi-fluid mud was subjected to an increased pressure of 3⅓ tons, or, making all allowance for friction and rigidity of strata above, the augmented pressure resulting would be at least double the normal pressure.

If the "pocket" was small, the resulting motion might at first be much slower than subsequently, when its enveloping surface-area became increased by penetration between the outlying gravel and the rock.

The peculiarity of the case, that a very slight settlement would bring a lateral thrust to bear on piles, the resistance of which to this kind of pressure was only the soft mud of the upper strata, seems to account for the ultimate acceleration succeeding to the slowness of the initial motion.

The case presents an almost isolated instance of the fall of a pile-supported structure, accompanied with the *overturning of the piles*. In this aspect it deserves the attention of engineers. I recall but one other instance, that of the sinking of a pier of the bridge of Tours (France). (See article "FOUNDATIONS," Johnson's Cyclopedia, by Prof. DeVolson

Wood, or the Annales des Ponts et Chaussées, 1839.) The cause (not given in the Cyclopedia article) appears to have been the want of lateral support (see the "Annales"), there being but about four feet of sand over the tufa in which the points of the piles rested, all above for four feet more upward, having been dredged away in the process of preparing the foundation, without subsequent thorough replacement. No cause in this case is given for the development of lateral pressure; but a structure on piles thus circumstanced is, as it were, "on stilts." This is a contingency in the use of piles which deserves more attention than it has hitherto received.

The *general* character ("treacherous and unstable") of the stratification (South Street Bridge) being known, *i. e.*, mud and gravel overlying rock, *each* pile foundation of the approach became a subject, as Mr. Stauffer correctly remarks, which demanded thorough examination. Alluvial and drift foundations, like this, are notorious for their variability and for the occurrence of intrusions of clay or mud.

The "pointed iron rod" would almost certainly be stopped by the first hard gravel stratum. Failing this adequate examination, and resorting to piles as a specific for the unreliableness of the subsoil, the failure to shoe and band the piles was, I think, another error. The weight-bearing measure of piles thus driven cannot be deduced with any certainty from observation of the ultimate penetration under the blow of a known "fall" and hammer-weight; so much living force is wasted in shattering the head, in crushing the toe, and in elastic compression of the pile itself, the unshod point stopping in hard gravel where a shod point would continue to penetrate. In the present case not only was unimpeded *penetration* desirable as furnishing the most reliable test of weight-bearing capacity, but, in uncertainty as to stratification, it was desirable to reach, if not the rock, the greatest possible depth.

Finally, this single case enforces attention to two important matters concerning pile foundations on treacherous soils hitherto, I believe, almost unnoticed. First. Well ascertained and ample weight-bearing capacity of *the piles themselves* is not alone proof that the foundation is stable. We must know *what lies below* the particular stratification in which the piles terminate. Second. We must pay so much attention to what *lies above* the same stratification as to assure ourselves that under all probable assumptions as to the way in which it may be called into play there shall be *ample lateral support*. In the case before us it seems

probable that the unequal settlement, *alone* due to the mud-pocket underlying one end of the pier, would not have caused the fall. The rotation that ensued was, by the great lateral rigidity of the bridge-floor, shifted from the lower extreme end of the foundation, where mere settlement would have placed it, to a point high above, causing thereby a powerful lateral (southward) thrust on the pile heads.

The case of Fort Livingston, La., has some interest in this connection. The work, intended to defend the entrance to Barataria Bay (from which the Mississippi River banks opposite and below New Orleans may be reached in numerous localities) is on the western extremity of a sand island (Grand Terre). The shore foundation, for a long ways west of the mouth, is made up of the sedimentary discharge of the river, the coarser materials of which, of a peculiarly fine-grained, dark-colored sand, are thrown up along the shores. The shore margins of all these islands are of this sand—hard to the foot, and though of very fine and dark-colored sand, yet otherwise quite similar to the white quartzose sand margins of the Atlantic and Gulf shores of Florida.

Before commencing the foundations a single boring was made. It showed sand to a depth of nearly 20 feet, underneath which, by an abrupt transition, was a slate-colored clay, very soft and plastic as brought up, but hardening by drying. The belief was at first entertained that so thick a stratum of sand would support the work without material settlement. In this we were disappointed. No considerable weight could be applied anywhere without producing a sensible settlement. Borings were then extended everywhere over the site, varying in their results from the first, only in showing a diminution of thickness of the sand stratum on receding from the shore—in the more remote trial pits, to 11 or 12 feet.

The inference was drawn that the settlement was due to the displacement of the very plastic underlying clay. Had it been a question of sustaining an isolated weight (as *e. g.*, a bridge pier), the resort to artificial aids might, perhaps, have been entertained. But while the settlement of the masonry (scarp and counter-scarp walls, casemates, &c., &c.), was the only thing to be deprecated, the weight of any part of these, as applied to its own foundations, formed no rule by which to estimate total effect, since the whole formed an integral which carried down, as a floor, the underlying sand stratum. Moreover, the weight of the masonry was but a fractional part of the total weight imposed on this floor ; the adja-

cent embankment (glacis, ramparts, &c.) weighing many times more than the masonry. Hence, it was believed that nothing would be gained by using piles or grillages.

Unquestionably the best precaution would have been to load all the foundations, and, before building the superstructures, to have also imposed, at least to such extent as might be practicable, the sand embankments, and await the result. To a certain degree this was actually practised; but under the belief that no very serious harm would result, and from the desirableness of continuing to completion a work in an out of the way place where an organized laboring force could not be readily discharged and brought together again, the final result of this process was not waited for.

The work was essentially finished about the year 1850, and although comprising quite an extent of casemate arches, and a very long arched counter-scarp gallery, it is stated that no serious damage has since occurred. While the operations of loading and subsequent building were going on, settlements of as much as two or three feet had been exhibited. Since that it is not improbable that there has been generally a further settlement of one or two feet. Although there was *inequality* of settlement (such as the middle portions of the scarp walls going down more than the corners), there was never exhibited an abrupt transition, and hence the exemption from dislocations of the masonry. In one case, by way of experiment, a portion of the commenced scarp wall, only two or three feet high, was loaded with the full weight it was ultimately to (directly) bear, while its prolongation was left without load. Settlement followed, but it was extended for a considerable distance along the unloaded part, so that no crack resulted.

The case of Fort Livingston has little analogy with that of the South Street Bridge; the only features in common being the existence (under very different circumstances) of a soft plastic stratum under the one supposed to be fully capable of weight-bearing. Both cases, however, illustrate effects which may ensue from the existence of soft or plastic strata beneath the hard and compressed ones, on which a structure is immediately to rest.

AMERICAN SOCIETY OF CIVIL ENGINEERS.

INSTITUTED 1852.

TRANSACTIONS.

NOTE.—This Society is not responsible, as a body, for the facts and opinions advanced in any of its publications.

CCII.

Vol. IX.—August, 1880.

NOTE ON KUTTER'S DIAGRAM.

By CHARLES H. SWAN, C. E., Member of the Society.

The author of the paper* on "The Flow of Water in Small Channels," has omitted the proof that the diagram of Plate 1, Vol. viii, correctly represents the co-efficients of Kutter's formula. It is believed, however, that a concise proof that this diagram rigorously represents the formula, and is in no respect an approximation, would greatly increase confidence in its use.

Kutter's formula is a complicated equation for obtaining the value of the velocity co-efficient c, in the well known hydraulic formula,

$$v = c\sqrt{Rs} \tag{1}$$

Kutter's equation, in its most general form, is

$$c = \frac{y}{1 + \frac{x}{\sqrt{R}}}, \tag{2}$$

from which may be deduced the proportion

$$\frac{c}{\sqrt{R}} = \frac{y}{x + \sqrt{R}}. \tag{3}$$

The diagram of Plate 1, Vol. viii, is simply a graphical construction of this proportion, by means of similar triangles, and is, therefore, strictly accurate.

* The Flow of Water in Small Channels. R. Hering. Vol. viii, p. 1, January, 1879.

The variables x and y, being used in the diagram as co-ordinates, determine the loci of n and s, of which they both are functions.

The fact that the loci of n are straight lines radiating from a fixed point on the axis of abscissas, is of great value in plotting the diagram on a large scale. It may be proved thus :

The value of x is essentially negative with reference to the co-ordinate axes of the diagram. The equations of x and y may, therefore, be written—

$$x = -\left(a + \frac{m}{s}\right) n \tag{4}$$

$$y = a + \frac{m}{s} + \frac{l}{n} \tag{5}$$

$$\text{from which, } y = \frac{l}{n} - \frac{x}{n} \tag{6}$$

This becomes an equation of the first degree when n is constant, and indicates that the loci of n are straight lines.

The point of intersection of a locus of n with the axis of abscissas may be obtained by making $y = o$ in equation (6), giving

$$x = l \tag{7}$$

This equation is independent of the value of n, and is positive. All loci of n will therefore intersect the axis of abscissas at one point, at the right of the origin, and distant therefrom an amount equal to the numerical value of l.

For measures in meters $l = 1$, and as x and $\sqrt{R}$ are plotted to the same scale, the center of radiation is at the point where

$$\sqrt{R} = \text{unity, or 1 meter.}$$

The exact character of the curves forming the loci of s is of less consequence, but may be thus shown.

When s is constant, equations (4) and (5) may be written

$$x = -bn \tag{8}$$

$$y = b + \frac{l}{n} \tag{9}$$

Referring y to a new axis of abscissas, parallel to the original one, and distant from it an amount equal to $b = a + \frac{m}{s}$, we have for the new value of y

$$y_1 = \frac{l}{n} \tag{10}$$

combining equations (8) and (10), we have the equation of the loci of s

$$x\, y\, y_1 = -b\, l \text{ (a constant)} \tag{11}$$

This is the equation of an hyperbola referred to its center and asymptotes, which being at right angles to each other, by construction, cause the loci of s to be equilateral hyperbolas having the axis of ordinates as a common asymptote. The asymptote parallel to the axis of abscissas has a different position for each curve, determined by the equation

$$y_2 = a + \frac{m}{s}. \qquad (12)$$

The annexed table of co-ordinates was prepared by the writer while making a draught of Kutter's diagram, on a large scale, for the City Engineer's office at Providence, R. I. It contains values of x and y, for measures in *meters*, which may be used for extending the range of Plate 1.

The metric values of x and y, given in this table, may be plotted on a diagram adapted to measures in English feet, by the following simple process.

The scales for plotting the values of x, y, $\sqrt{R}$, and c, having been determined for measures in feet, prepare two auxiliary scales, upon separate strips of paper, by which to plot the metric values of x and y.

The scale for metric values of x is made by marking upon one of the strips, the length corresponding to $x = 1.8113$ at the scale chosen for plotting x for measures in *feet*. This length is to be divided into 100 equal parts and numbered consecutively from 0. to 1., forming a scale of hundredths.

The scale for metric values of y is made by marking upon the other strip, the length corresponding to $y = 181.13$ at the scale chosen for plotting y for measures in *feet*. This length is to be divided into 100 equal parts and numbered consecutively from 0. to 100., forming a scale of units.

The writer has also prepared a series of diagrams for directly determining the dimensions of sewers, based upon the formula of Ganguillet and Kutter.

Copies of the most important diagrams are herewith presented.

NOTE.—These copies are deposited at the Rooms of the Society.

Table of Co-ordinates.

Values, for measures in meters, of x and y in the equati

S	n = 0.008		n = 0.009		n = 0.010		n = 0.011		n = 0.012		n = 0.013		n = 0.014		n = 0.015		n = 0.016		n = 0.017		n = 0.018		
	y	x	y	x	y	x	y	x	y	x	y	x	y	x	y	x	y	x	y	x	y	x	
0.000025											161.92	1.1050	156.43	1.1900	151.67	1.2750							
0.000030							165.58	0.8213	158.00	0.8960	151.59	0.9707	146.10	1.0453	141.33	1.1200	137.17	1.1947	133.49	1.2693			
0.000035							158.19	0.7401	150.62	0.8074	144.21	0.8747	138.71	0.9420	133.95	1.0093	129.79	1.0766	126.11	1.1439	122.84	1.2118	1
0.000040					161.75	0.6175	152.66	0.6792	145.08	0.7410	138.67	0.8027	133.18	0.8645	128.42	0.9263	124.25	0.9880	120.57	1.0497	117.31	1.1115	1
0.000045					157.44	0.5744	148.35	0.6319	140.78	0.6893	134.37	0.7468	128.87	0.8042	124.11	0.8617	119.94	0.9191	116.27	0.9765	113.00	1.0340	1
0.000050					154.00	0.5400	144.91	0.5940	137.33	0.6480	130.92	0.7020	125.43	0.7560	120.67	0.8100	116.50	0.8640	112.82	0.9180	109.56	0.9720	1
0.000055					151.18	0.5118	142.09	0.5630	134.51	0.6142	128.10	0.6654	122.61	0.7165	117.85	0.7677	113.68	0.8189	110.00	0.8701	106.74	0.9213	1
0.000060			159.94	0.4385	148.83	0.4883	139.74	0.5372	132.17	0.5860	125.76	0.6348	120.26	0.6837	115.50	0.7325	111.33	0.7813	107.66	0.8301	104.39	0.8790	1
0.000065			157.96	0.4216	146.85	0.4685	137.75	0.5153	130.18	0.5621	123.77	0.6090	118.27	0.6558	113.51	0.7027	109.35	0.7495	105.67	0.7964	102.40	0.8432	
0.000070			156.25	0.4063	145.14	0.4514	136.05	0.4966	128.48	0.5417	122.07	0.5869	116.57	0.6320	111.81	0.6771	107.64	0.7223	103.97	0.7674	100.70	0.8126	
0.000075			154.78	0.3930	143.67	0.4367	134.58	0.4803	127.00	0.5240	120.59	0.5677	115.10	0.6113	110.33	0.6550	106.17	0.6987	102.49	0.7423	99.22	0.7860	
0.000080			153.49	0.3814	142.37	0.4237	133.28	0.4661	125.71	0.5085	119.30	0.5509	113.80	0.5933	109.04	0.6356	104.87	0.6770	101.19	0.7204	97.93	0.7628	
0.000085			152.35	0.3711	141.23	0.4123	132.14	0.4536	124.57	0.4948	118.16	0.5361	112.66	0.5773	107.90	0.6185	103.73	0.6598	100.06	0.7010	96.79	0.7422	
0.000090			151.33	0.3620	140.22	0.4022	131.13	0.4424	123.56	0.4827	117.14	0.5229	111.65	0.5631	106.89	0.6033	102.72	0.6436	99.04	0.6838	95.78	0.7240	
0.000095			150.43	0.3538	139.32	0.3932	130.22	0.4325	122.65	0.4718	116.24	0.5111	110.74	0.5504	105.98	0.5897	101.82	0.6291	98.14	0.6684	94.87	0.7077	
0.00010	163.50	0.3080	149.61	0.3435	138.50	0.3850	129.41	0.4235	121.83	0.4620	115.42	0.5005	109.93	0.5390	105.17	0.5775	101.00	0.6160	97.32	0.6545	94.06	0.6930	
0.00012	160.92	0.2873	147.03	0.3233	135.92	0.3592	126.83	0.3951	119.25	0.4310	112.84	0.4669	107.35	0.5028	102.58	0.5388	98.42	0.5747	94.74	0.6106	91.47	0.6465	
0.00015	158.33	0.2667	144.44	0.3000	133.33	0.3333	124.24	0.3667	116.67	0.4000	110.26	0.4333	104.76	0.4667	100.00	0.5000	95.83	0.5333	92.16	0.5667	88.89	0.6000	
0.0002	155.75	0.2460	141.86	0.2767	130.75	0.3075	121.66	0.3382	114.08	0.3690	107.67	0.3997	102.18	0.4305	97.42	0.4613	93.25	0.4920	89.57	0.5227	86.31	0.5535	
0.0003	153.17	0.2253	139.29	0.2535	128.17	0.2817	119.08	0.3098	111.50	0.3380	105.09	0.3662	99.60	0.3943	94.83	0.4225	90.67	0.4507	86.99	0.4788	83.72	0.5070	
0.0004	151.87	0.2150	137.99	0.2419	126.87	0.2687	117.78	0.2956	110.21	0.3225	103.80	0.3494	98.30	0.3763	93.54	0.4031	89.37	0.4300	85.70	0.4569	82.43	0.4838	
0.0005	151.10	0.2088	137.21	0.2349	126.10	0.2610	117.01	0.2871	109.43	0.3132	103.02	0.3393	97.53	0.3654	92.77	0.3915	88.60	0.4176	84.92	0.4437	81.66	0.4698	
0.0006	150.58	0.2047	136.69	0.2302	125.58	0.2558	116.49	0.2814	108.92	0.3070	102.51	0.3326	97.01	0.3582	92.25	0.3837	88.08	0.4093	84.41	0.4349	81.14	0.4605	
0.0007	150.21	0.2017	136.32	0.2269	125.21	0.2521	116.12	0.2774	108.55	0.3026	102.14	0.3278	96.64	0.3530	91.88	0.3782	87.71	0.4034	84.04	0.4286	80.77	0.4539	
0.0008	149.94	0.1995	136.05	0.2244	124.94	0.2494	115.85	0.2743	108.27	0.2992	101.86	0.3242	96.37	0.3491	91.60	0.3741	87.44	0.3990	83.76	0.4239	80.49	0.4489	
0.0009	149.72	0.1978	135.83	0.2225	124.72	0.2472	115.63	0.2719	108.06	0.2967	101.64	0.3214	96.15	0.3461	91.39	0.3708	87.22	0.3956	83.54	0.4203	80.28	0.4450	
0.0010	149.55	0.1964	135.66	0.2210	124.55	0.2455	115.46	0.2700	107.88	0.2946	101.47	0.3191	95.98	0.3437	91.22	0.3683	87.05	0.3929	83.37	0.4173	80.11	0.4419	
0.0020	148.77	0.1902	134.89	0.2140	123.77	0.2377	114.68	0.2615	107.11	0.2853	100.70	0.3091	95.20	0.3329	90.44	0.3566	86.27	0.3804	82.60	0.4042	79.33	0.4280	
0.0040	148.39	0.1871	134.50	0.2105	123.39	0.2339	114.30	0.2573	106.72	0.2806	100.31	0.3040	94.82	0.3274	90.05	0.3508	85.89	0.3742	82.21	0.3976	78.94	0.4210	
0.0060	148.26	0.1861	134.37	0.2093	123.26	0.2326	114.17	0.2558	106.59	0.2791	100.18	0.3024	94.69	0.3256	89.92	0.3489	85.76	0.3721	82.08	0.3954	78.81	0.4187	
0.0080	148.19	0.1856	134.30	0.2087	123.19	0.2319	114.10	0.2551	106.53	0.2783	100.12	0.3015	94.62	0.3247	89.86	0.3479	85.69	0.3711	82.02	0.3943	78.75	0.4175	
0.0100	148.15	0.1852	134.26	0.2083	123.15	0.2315	114.06	0.2546	106.48	0.2778	100.07	0.3009	94.58	0.3241	89.82	0.3473	85.65	0.3704	81.97	0.3935	78.71	0.4167	
0.1000	148.01	0.1841	134.13	0.2071	123.01	0.2301	113.92	0.2532	106.35	0.2762	99.94	0.2992	94.44	0.3222	89.68	0.3452	85.51	0.3682	81.84	0.3912	78.57	0.4143	
1.0000	148.00	0.1840	134.11	0.2070	123.00	0.2300	113.91	0.2530	106.33	0.2760	99.92	0.2990	94.43	0.3220	89.67	0.3450	85.50	0.3680	81.82	0.3910	78.56	0.4140	

S	n = 0.025		n = 0.026		n = 0.027		n = 0.028		n = 0.029		n = 0.030		n = 0.031		n = 0.032		n = 0.033		n = 0.034		n = 0.035		
	y	x	y	x	y	x	y	x	y	x	y	x	y	x	y	x	y	x	y	x	y	x	
0.000060	88.83	1.2208																					
0.000065	86.85	1.1711	85.31	1.2180																			
0.000070	85.14	1.1286	83.60	1.1737	82.18	1.2189																	
0.000075	83.67	1.0917	82.13	1.1354	80.70	1.1790	79.38	1.2227															
0.000080	82.37	1.0594	80.84	1.1018	79.41	1.1441	78.09	1.1865	76.86	1.2289													
0.000085	81.23	1.0309	79.70	1.0721	78.27	1.1133	76.95	1.1546	75.72	1.1958	74.51	1.2370											
0.000090	80.22	1.0055	78.68	1.0458	77.26	1.0860	75.94	1.1262	74.70	1.1664	73.56	1.2066	72.48	1.2469									

Table of Co-ordinates.

...es, for measures in meters, of x and y in the equation $c = \frac{y}{1 + \frac{x}{\sqrt{R}}}$.

n = 0.012		n = 0.013		n = 0.014		n = 0.015		n = 0.016		n = 0.017		n = 0.018		n = 0.019		n = 0.020		n = 0.021		n = 0.022		n = 0.023		n = 0.024	
y	x	y	x	y	x	y	x	y	x	y	x	y	x	y	x	y	x	y	x	y	x	y	x	y	x
		161.92	1.1050	156.43	1.1900	151.67	1.2750																		
158.00	0.8960	151.59	0.9707	146.10	1.0453	141.33	1.1200	137.17	1.1947	133.49	1.2693														
150.62	0.8014	144.21	0.8747	138.71	0.9420	133.95	1.0093	129.79	1.0766	126.11	1.1439	122.84	1.2118	119.92	1.2754										
146.08	0.7410	138.67	0.9027	133.15	0.8645	128.42	0.9263	124.25	0.9880	120.57	1.0491	117.31	1.1115	114.38	1.1732	111.75	1.2350								
140.78	0.6893	134.37	0.7468	128.87	0.8042	124.11	0.8617	119.94	0.9191	116.27	0.9765	113.00	1.0340	110.07	1.0914	107.44	1.1489	105.06	1.2063	102.90	1.2638				
137.33	0.6480	130.92	0.7020	125.43	0.7560	120.67	0.8100	116.50	0.8640	112.82	0.9180	109.56	0.9720	106.63	1.0260	104.00	1.0800	101.62	1.1340	99.45	1.1880	97.48	1.2420		
134.51	0.6142	128.10	0.6654	122.61	0.7165	117.85	0.7677	113.68	0.8189	110.00	0.8701	106.74	0.9213	103.81	0.9724	101.18	1.0236	98.80	1.0748	96.64	1.1260	94.66	1.1772	92.85	1.2284
132.17	0.5860	125.76	0.6348	120.26	0.6837	115.50	0.7325	111.33	0.7813	107.66	0.8301	104.39	0.8790	101.46	0.9278	98.93	0.9767	96.45	1.0255	94.29	1.0743	92.31	1.1232	90.50	1.1720
130.18	0.5621	123.77	0.6090	118.27	0.6558	113.51	0.7027	109.35	0.7495	105.67	0.7964	102.40	0.8432	99.48	0.8901	96.85	0.9369	94.46	0.9838	92.30	1.0306	90.32	1.0775	88.51	1.1243
128.48	0.5417	122.07	0.5869	116.57	0.6320	111.81	0.6771	107.64	0.7223	103.97	0.7674	100.70	0.8126	97.77	0.8577	95.14	0.9029	92.76	0.9480	90.60	0.9932	88.62	1.0383	86.81	1.0834
127.00	0.5240	120.59	0.5677	115.10	0.6113	110.33	0.6550	106.17	0.6987	102.49	0.7423	99.22	0.7860	96.30	0.8297	93.67	0.8733	91.29	0.9170	89.12	0.9607	87.14	1.0043	85.33	1.0480
125.71	0.5085	119.30	0.5509	113.80	0.5933	109.04	0.6356	104.87	0.6770	101.19	0.7204	97.93	0.7628	95.01	0.8051	92.37	0.8475	89.99	0.8899	87.83	0.9323	85.85	0.9746	84.04	1.0170
124.57	0.4948	118.16	0.5361	112.66	0.5773	107.90	0.6185	103.73	0.6598	100.06	0.7010	96.79	0.7422	93.87	0.7835	91.23	0.8247	88.85	0.8659	86.69	0.9072	84.71	0.9484	82.90	0.9896
123.56	0.4827	117.14	0.5229	111.65	0.5631	106.89	0.6033	102.72	0.6436	99.04	0.6838	95.78	0.7240	92.85	0.7642	90.22	0.8044	87.84	0.8447	85.68	0.8849	83.70	0.9251	81.89	0.9653
122.65	0.4718	116.24	0.5111	110.74	0.5504	105.98	0.5897	101.82	0.6291	98.14	0.6684	94.87	0.7077	91.95	0.7470	89.32	0.7863	86.93	0.8256	84.77	0.8650	82.79	0.9043	80.98	0.9436
121.83	0.4620	115.42	0.5005	109.93	0.5390	105.17	0.5775	101.00	0.6160	97.32	0.6545	94.06	0.6930	91.13	0.7315	88.50	0.7700	86.12	0.8085	83.95	0.8470	81.98	0.8855	80.17	0.9240
119.25	0.4310	112.84	0.4669	107.35	0.5028	102.58	0.5388	98.42	0.5747	94.74	0.6106	91.47	0.6465	88.55	0.6824	85.92	0.7183	83.54	0.7543	81.37	0.7902	79.39	0.8261	77.58	0.8620
116.67	0.4000	110.26	0.4333	104.76	0.4667	100.00	0.5000	95.83	0.5333	92.16	0.5667	88.89	0.6000	85.96	0.6333	83.33	0.6667	80.95	0.7000	78.79	0.7333	76.81	0.7667	75.00	0.8000
114.08	0.3690	107.67	0.3997	102.18	0.4305	97.42	0.4613	93.25	0.4920	89.57	0.5227	86.31	0.5535	83.38	0.5842	80.75	0.6150	78.37	0.6457	76.20	0.6765	74.23	0.7072	72.42	0.7380
111.50	0.3380	105.09	0.3662	99.60	0.3943	94.83	0.4225	90.67	0.4507	86.99	0.4788	83.72	0.5070	80.80	0.5352	78.17	0.5633	75.79	0.5915	73.62	0.6197	71.64	0.6478	69.83	0.6760
110.21	0.3225	103.80	0.3494	98.30	0.3763	93.54	0.4031	89.37	0.4300	85.70	0.4569	82.43	0.4838	79.51	0.5106	76.87	0.5375	74.49	0.5644	72.33	0.5913	70.35	0.6181	68.54	0.6450
109.43	0.3132	103.02	0.3393	97.53	0.3654	92.77	0.3915	88.60	0.4176	84.92	0.4437	81.66	0.4698	78.73	0.4959	76.10	0.5220	73.72	0.5481	71.56	0.5742	69.59	0.6003	67.77	0.6264
108.92	0.3070	102.51	0.3326	97.01	0.3582	92.25	0.3837	88.08	0.4093	84.41	0.4349	81.14	0.4605	78.21	0.4861	75.58	0.5117	73.20	0.5372	71.04	0.5628	69.06	0.5884	67.25	0.6140
108.55	0.3026	102.14	0.3278	96.64	0.3530	91.88	0.3782	87.71	0.4034	84.04	0.4286	80.77	0.4539	77.84	0.4791	75.21	0.5043	72.83	0.5295	70.67	0.5547	68.69	0.5799	66.88	0.6051
108.27	0.2992	101.86	0.3242	96.37	0.3491	91.60	0.3741	87.44	0.3990	83.76	0.4239	80.49	0.4489	77.57	0.4738	74.94	0.4987	72.56	0.5237	70.39	0.5486	68.41	0.5735	66.60	0.5985
108.06	0.2967	101.64	0.3214	96.15	0.3461	91.39	0.3708	87.22	0.3956	83.54	0.4203	80.28	0.4450	77.35	0.4697	74.72	0.4944	72.34	0.5192	70.18	0.5439	68.20	0.5686	66.39	0.5933
107.88	0.2946	101.47	0.3191	95.98	0.3437	91.22	0.3683	87.05	0.3929	83.37	0.4173	80.11	0.4419	77.18	0.4664	74.55	0.4910	72.17	0.5156	70.00	0.5401	68.03	0.5646	66.22	0.5892
107.11	0.2853	100.70	0.3091	95.20	0.3329	90.44	0.3566	86.27	0.3804	82.60	0.4042	79.33	0.4280	76.41	0.4517	73.77	0.4755	71.39	0.4993	69.23	0.5231	67.25	0.5468	65.44	0.5706
106.72	0.2806	100.31	0.3040	94.82	0.3274	90.05	0.3508	85.89	0.3742	82.21	0.3976	78.94	0.4210	76.02	0.4443	73.39	0.4677	71.01	0.4911	68.84	0.5145	66.86	0.5379	65.05	0.5613
106.59	0.2791	100.18	0.3024	94.69	0.3256	89.92	0.3489	85.76	0.3721	82.08	0.3954	78.81	0.4187	75.89	0.4419	73.26	0.4652	70.88	0.4884	68.71	0.5117	66.74	0.5349	64.92	0.5582
106.53	0.2783	100.12	0.3015	94.62	0.3247	89.86	0.3479	85.69	0.3711	82.02	0.3943	78.75	0.4175	75.82	0.4407	73.19	0.4639	70.81	0.4871	68.65	0.5103	66.67	0.5335	64.86	0.5567
106.48	0.2778	100.07	0.3009	94.58	0.3241	89.82	0.3473	85.65	0.3704	81.97	0.3935	78.71	0.4167	75.78	0.4398	73.15	0.4630	70.77	0.4861	68.60	0.5093	66.63	0.5324	64.82	0.5556
106.35	0.2762	99.94	0.2992	94.44	0.3222	89.68	0.3452	85.51	0.3682	81.84	0.3912	78.57	0.4143	75.65	0.4373	73.01	0.4603	70.63	0.4833	68.47	0.5063	66.49	0.5293	64.68	0.5524
106.33	0.2760	99.92	0.2990	94.43	0.3220	89.67	0.3450	85.50	0.3680	81.82	0.3910	78.56	0.4140	75.63	0.4370	73.00	0.4600	70.62	0.4830	68.46	0.5060	66.48	0.5290	64.67	0.5520

n = 0.029		n = 0.030		n = 0.031		n = 0.032		n = 0.033		n = 0.034		n = 0.035		n = 0.036		n = 0.037		n = 0.038		n = 0.039		n = 0.040		n = 0.041	
y	x	y	x	y	x	y	x	y	x	y	x	y	x	y	x	y	x	y	x	y	x	y	x	y	x
76.86	1.2289																								
75.72	1.1958	74.57	1.2370																						

0.0010	149.55	0.1964	135.66	0.2210	124.55	0.2455	115.46	0.2701	107.88	0.2946	101.47	0.3191	95.98	0.3437	91.22	0.3683	87.05	0.3929	83.37	0.4173	80.11	0.4419	77.18
0.0020	148.77	0.1902	134.89	0.2140	123.77	0.2377	114.68	0.2615	107.11	0.2853	100.70	0.3091	95.20	0.3329	90.44	0.3566	86.27	0.3804	82.60	0.4042	79.33	0.4280	76.41
0.0040	148.39	0.1871	134.50	0.2105	123.39	0.2339	114.30	0.2573	106.72	0.2806	100.31	0.3040	94.82	0.3274	90.05	0.3508	85.89	0.3742	82.21	0.3976	78.94	0.4210	76.02
0.0060	148.26	0.1861	134.37	0.2093	123.26	0.2326	114.17	0.2559	106.59	0.2791	100.18	0.3024	94.69	0.3256	89.92	0.3489	85.76	0.3721	82.08	0.3954	78.81	0.4187	75.89
0.0080	148.19	0.1856	134.30	0.2087	123.19	0.2319	114.10	0.2551	106.53	0.2783	100.12	0.3015	94.62	0.3247	89.86	0.3479	85.69	0.3711	82.02	0.3943	78.75	0.4175	75.82
0.0100	148.15	0.1852	134.26	0.2083	123.15	0.2315	114.06	0.2546	106.48	0.2778	100.07	0.3009	94.58	0.3241	89.82	0.3473	85.65	0.3704	81.97	0.3935	78.71	0.4167	75.78
0.1000	148.01	0.1841	134.13	0.2071	123.01	0.2301	113.92	0.2532	106.35	0.2762	99.94	0.2992	94.44	0.3222	89.68	0.3452	85.51	0.3682	81.84	0.3912	78.57	0.4143	75.65
1.0000	148.00	0.1840	134.11	0.2070	123.00	0.2300	113.91	0.2530	106.33	0.2760	99.92	0.2990	94.43	0.3220	89.67	0.3450	85.50	0.3680	81.82	0.3910	78.56	0.4140	75.63

S	n = 0.025		n = 0.026		n = 0.027		n = 0.028		n = 0.029		n = 0.030		n = 0.031		n = 0.032		n = 0.033		n = 0.034		n = 0.035		n = 0.
	y	x	y	x	y	x	y	x	y	x	y	x	y	x	y	x	y	x	y	x	y	x	y
0.000060	88.83	1.2208																					
0.000065	86.85	1.1711	85.31	1.2180																			
0.000070	85.14	1.1286	83.60	1.1737	82.18	1.2189																	
0.000075	83.67	1.0917	82.13	1.1354	80.70	1.1790	79.38	1.2227															
0.000080	82.37	1.0594	80.84	1.1018	79.41	1.1441	78.09	1.1866	76.86	1.2289													
0.000085	81.23	1.0309	79.70	1.0721	78.27	1.1133	76.95	1.1546	75.72	1.1958	74.57	1.2370											
0.000090	80.22	1.0055	78.68	1.0458	77.26	1.0860	75.94	1.1262	74.70	1.1664	73.56	1.2066	72.48	1.2469									
0.000095	79.32	0.9829	77.78	1.0222	76.35	1.0615	75.03	1.1008	73.80	1.1402	72.65	1.1795	71.57	1.2188	70.57	1.2581							
0.00010	78.50	0.9625	76.96	1.0010	75.54	1.0395	74.21	1.0779	72.98	1.1165	71.83	1.1550	70.76	1.1935	69.75	1.2320	68.80	1.2705					
0.00012	75.92	0.8979	74.38	0.9339	72.95	0.9698	71.63	1.0057	70.40	1.0416	69.25	1.0775	68.17	1.1134	67.17	1.1493	66.22	1.1853	65.33	1.2212	64.49	1.2571	
0.00015	73.33	0.8333	71.79	0.8667	70.37	0.9000	69.05	0.9333	67.82	0.9667	66.67	1.0000	65.59	1.0333	64.58	1.0667	63.64	1.1000	62.75	1.1333	61.90	1.1666	61.11
0.0002	70.75	0.7687	69.21	0.7995	67.79	0.8302	66.46	0.8610	65.23	0.8918	64.08	0.9225	63.01	0.9532	62.00	0.9840	61.05	1.0147	60.16	1.0455	59.32	1.0762	58.53
0.0003	68.17	0.7042	66.63	0.7324	65.20	0.7605	63.88	0.7887	62.65	0.8168	61.50	0.8450	60.42	0.8732	59.42	0.9013	58.47	0.9295	57.58	0.9577	56.74	0.9858	55.94
0.0004	66.87	0.6719	65.34	0.6998	63.91	0.7256	62.59	0.7525	61.36	0.7794	60.21	0.8062	59.13	0.8331	58.12	0.8600	57.18	0.8869	56.29	0.9138	55.45	0.9406	54.65
0.0005	66.10	0.6525	64.56	0.6786	63.14	0.7047	61.81	0.7308	60.58	0.7569	59.43	0.7830	58.36	0.8090	57.35	0.8352	56.40	0.8613	55.51	0.8874	54.67	0.9135	53.88
0.0006	65.58	0.6396	64.04	0.6652	62.62	0.6907	61.30	0.7163	60.07	0.7419	58.92	0.7675	57.84	0.7931	56.83	0.8187	55.89	0.8442	54.99	0.8698	54.15	0.8954	53.36
0.0007	65.21	0.6304	63.68	0.6556	62.25	0.6808	60.93	0.7060	59.70	0.7312	58.55	0.7564	57.47	0.7816	56.46	0.8068	55.52	0.8321	54.63	0.8573	53.78	0.8825	52.99
0.0008	64.94	0.6234	63.40	0.6484	61.97	0.6733	60.65	0.6982	59.42	0.7232	58.27	0.7481	57.20	0.7730	56.19	0.7980	55.24	0.8229	54.35	0.8479	53.51	0.8728	52.71
0.0009	64.72	0.6180	63.18	0.6428	61.76	0.6675	60.44	0.6921	59.20	0.7169	58.05	0.7416	56.98	0.7664	55.97	0.7911	55.02	0.8158	54.13	0.8406	53.29	0.8653	52.50
0.0010	64.55	0.6137	63.01	0.6383	61.59	0.6629	60.26	0.6874	59.03	0.7120	57.88	0.7365	56.81	0.7611	55.80	0.7856	54.85	0.8101	53.96	0.8347	53.12	0.8592	52.33
0.0020	63.77	0.5944	62.24	0.6182	60.81	0.6419	59.49	0.6657	58.26	0.6895	57.11	0.7132	56.03	0.7370	55.02	0.7608	54.08	0.7846	53.19	0.8084	52.35	0.8321	51.55
0.0040	63.39	0.5847	61.85	0.6081	60.42	0.6314	59.10	0.6548	57.87	0.6782	56.72	0.7016	55.64	0.7250	54.64	0.7484	53.69	0.7718	52.80	0.7952	51.96	0.8195	51.16
0.0060	63.26	0.5814	61.72	0.6047	60.29	0.6280	58.97	0.6512	57.74	0.6745	56.59	0.6977	55.52	0.7210	54.51	0.7443	53.56	0.7675	52.67	0.7908	51.83	0.8140	51.04
0.0080	63.19	0.5798	61.66	0.6031	60.23	0.6262	58.91	0.6494	57.68	0.6726	56.53	0.6958	55.45	0.7190	54.44	0.7422	53.50	0.7654	52.61	0.7886	51.76	0.8118	50.97
0.0100	63.15	0.5787	61.61	0.6019	60.19	0.6250	58.86	0.6482	57.63	0.6714	56.48	0.6945	55.41	0.7176	54.40	0.7408	53.45	0.7639	52.56	0.7871	51.72	0.8102	50.93
0.1000	63.01	0.5754	61.48	0.5984	60.05	0.6214	58.73	0.6444	57.50	0.6674	56.35	0.6904	55.27	0.7135	54.26	0.7365	53.32	0.7595	52.43	0.7825	51.59	0.8055	50.79
1.0000	63.00	0.5750	61.46	0.5980	60.04	0.6210	58.71	0.6440	57.48	0.6670	56.33	0.6900	55.26	0.7130	54.25	0.7360	53.30	0.7590	52.41	0.7820	51.57	0.8050	50.78

S	n = 0.042		n = 0.043		n = 0.044		n = 0.045		n = 0.046		n = 0.047		n = 0.048		n = 0.049		n = 0.050	
	y	x	y	x	y	x	y	x	y	x	y	x	y	x	y	x	y	x
0.0003	51.98	1.1830	51.42	1.2112	50.89	1.2393												
0.0004	50.68	1.1288	50.13	1.1556	49.60	1.1825	49.10	1.2094	48.61	1.2362	48.15	1.2631						
0.0005	49.91	1.0962	49.36	1.1223	48.83	1.1484	48.32	1.1745	47.84	1.2006	47.38	1.2267	46.93	1.2528				
0.0006	49.39	1.0745	48.84	1.1001	48.31	1.1256	47.80	1.1512	47.32	1.1768	46.86	1.2024	46.42	1.2280	45.99	1.2536		
0.0007	49.02	1.0590	48.47	1.0842	47.94	1.1094	47.44	1.1346	46.95	1.1598	46.49	1.1851	46.05	1.2103	45.62	1.2355		
0.0008	48.75	1.0474	48.19	1.0723	47.66	1.0972	47.16	1.1222	46.68	1.1471	46.21	1.1721	45.77	1.1970	45.34	1.2219	44.94	1.2468
0.0009	48.53	1.0383	47.98	1.0631	47.45	1.0878	46.94	1.1125	46.46	1.1372	46.00	1.1620	45.56	1.1866	45.13	1.2114	44.72	1.2361
0.0010	48.36	1.0311	47.81	1.0557	47.28	1.0802	46.77	1.1048	46.29	1.1293	45.83	1.1539	45.38	1.1784	44.96	1.2029	44.55	1.2275
0.0020	47.58	0.9986	47.03	1.0223	46.50	1.0461	46.00	1.0699	45.51	1.0936	45.05	1.1174	44.61	1.1412	44.18	1.1650	43.77	1.1887
0.0040	47.20	0.9823	46.64	1.0056	46.11	1.0290	45.61	1.0524	45.13	1.0758	44.66	1.0992	44.22	1.1226	43.79	1.1460	43.39	1.1693
0.0060	47.07	0.9769	46.51	1.0001	45.98	1.0233	45.48	1.0466	45.00	1.0699	44.53	1.0931	44.09	1.1164	43.67	1.1395	43.26	1.1629
0.0080	47.00	0.9742	46.45	0.9973	45.92	1.0205	45.42	1.0437	44.93	1.0669	44.47	1.0909	44.03	1.1133	43.60	1.1365	43.19	1.1597
0.0100	46.96	0.9723	46.41	0.9955	45.88	1.0186	45.37	1.0417	44.89	1.0649	44.43	1.0881	43.98	1.1112	43.56	1.1343	43.15	1.1575
0.1000	46.82	0.9666	46.27	0.9897	45.74	1.0126	45.24	1.0357	44.75	1.0587	44.29	1.0817	43.85	1.1047	43.42	1.1277	43.01	1.1507
1.0000	46.81	0.9661	46.26	0.9890	45.73	1.0120	45.22	1.0350	44.74	1.0580	44.28	1.0811	43.83	1.1040	43.41	1.1270	43.00	1.1500

y =

x =

7.11	0.2853	100.70	0.3091	95.20	0.3329	90.44	0.3566	86.27	0.3804	82.60	0.4042	79.33	0.4280	76.41	0.4517	73.77	0.4755	71.39	0.4993	69.23	0.5231	67.25	0.5468	65.44	0.5706
6.72	0.2806	100.31	0.3040	94.82	0.3274	90.05	0.3508	85.89	0.3742	82.21	0.3976	78.94	0.4210	76.02	0.4443	73.39	0.4677	71.01	0.4911	68.84	0.5145	66.86	0.5379	65.05	0.5613
6.59	0.2791	100.18	0.3024	94.69	0.3256	89.92	0.3489	85.76	0.3721	82.08	0.3954	78.81	0.4187	75.89	0.4419	73.26	0.4652	70.88	0.4884	68.71	0.5117	66.74	0.5349	64.92	0.5582
6.53	0.2783	100.12	0.3015	94.62	0.3247	89.86	0.3479	85.69	0.3711	82.02	0.3943	78.75	0.4175	75.82	0.4407	73.19	0.4639	70.81	0.4871	68.65	0.5103	66.67	0.5335	64.86	0.5567
6.48	0.2778	100.07	0.3009	94.58	0.3241	89.82	0.3473	85.65	0.3704	81.97	0.3935	78.71	0.4167	75.78	0.4398	73.15	0.4630	70.77	0.4861	68.60	0.5093	66.63	0.5324	64.82	0.5556
6.35	0.2762	99.94	0.2992	94.44	0.3222	89.68	0.3452	85.51	0.3682	81.84	0.3912	78.57	0.4143	75.65	0.4373	73.01	0.4603	70.63	0.4833	68.47	0.5063	66.49	0.5293	64.68	0.5524
6.33	0.2760	99.92	0.2990	94.43	0.3220	89.67	0.3450	85.50	0.3680	81.82	0.3910	78.56	0.4140	75.63	0.4370	73.00	0.4600	70.62	0.4830	68.46	0.5060	66.48	0.5290	64.67	0.5520

n = 0.029		n = 0.030		n = 0.031		n = 0.032		n = 0.033		n = 0.034		n = 0.035		n = 0.036		n = 0.037		n = 0.038		n = 0.039		n = 0.040		n = 0.041	
y	x	y	x	y	x	y	x	y	x	y	x	y	x	y	x	y	x	y	x	y	x	y	x	y	x
76.86	1.2289																								
75.72	1.1958	74.57	1.2370																						
74.70	1.1664	73.56	1.2066	72.48	1.2469																				
73.80	1.1402	72.65	1.1795	71.57	1.2188	70.57	1.2581																		
72.98	1.1165	71.83	1.1550	70.76	1.1935	69.75	1.2320	68.80	1.2705																
70.40	1.0416	69.25	1.0775	68.17	1.1134	67.17	1.1493	66.22	1.1853	65.33	1.2212	64.49	1.2571												
67.82	0.9667	66.67	1.0000	65.59	1.0333	64.58	1.0667	63.64	1.1000	62.75	1.1333	61.90	1.1666	61.11	1.2000	60.36	1.2333								
65.23	0.8918	64.08	0.9225	63.01	0.9532	62.00	0.9840	61.05	1.0147	60.16	1.0455	59.32	1.0762	58.53	1.1070	57.78	1.1377	57.01	1.1685	56.39	1.1992	55.75	1.2300		
62.65	0.8168	61.50	0.8450	60.42	0.8732	59.42	0.9013	58.47	0.9295	57.58	0.9577	56.74	0.9858	55.94	1.0140	55.19	1.0422	54.48	1.0704	53.81	1.0985	53.17	1.1267	52.56	1.1548
61.36	0.7794	60.21	0.8062	59.13	0.8331	58.12	0.8600	57.18	0.8869	56.29	0.9138	55.45	0.9406	54.65	0.9675	53.90	0.9944	53.19	1.0213	52.52	1.0481	51.87	1.0750	51.26	1.1019
60.58	0.7569	59.43	0.7830	58.36	0.8090	57.35	0.8352	56.40	0.8613	55.51	0.8874	54.67	0.9135	53.88	0.9396	53.13	0.9657	52.42	0.9918	51.74	1.0179	51.10	1.0440	50.49	1.0701
60.07	0.7419	58.92	0.7676	57.84	0.7931	56.83	0.8187	55.89	0.8442	54.99	0.8698	54.15	0.8954	53.36	0.9210	52.61	0.9466	51.90	0.9722	51.22	0.9977	50.58	1.0233	49.97	1.0489
59.70	0.7312	58.55	0.7564	57.47	0.7816	56.46	0.8068	55.52	0.8321	54.63	0.8573	53.78	0.8825	52.99	0.9077	52.24	0.9329	51.53	0.9581	50.85	0.9833	50.21	1.0086	49.60	1.0338
59.42	0.7232	58.27	0.7481	57.20	0.7730	56.19	0.7980	55.24	0.8229	54.35	0.8479	53.51	0.8728	52.71	0.8977	51.96	0.9227	51.25	0.9476	50.58	0.9725	49.94	0.9975	49.33	1.0224
59.20	0.7169	58.05	0.7416	56.98	0.7664	55.97	0.7911	55.02	0.8158	54.13	0.8406	53.29	0.8653	52.50	0.8900	51.75	0.9147	51.04	0.9394	50.36	0.9642	49.72	0.9889	49.11	1.0136
59.03	0.7120	57.88	0.7365	56.81	0.7611	55.80	0.7856	54.85	0.8101	53.96	0.8347	53.12	0.8592	52.33	0.8838	51.58	0.9083	50.87	0.9329	50.19	0.9575	49.55	0.9820	48.94	1.0065
58.26	0.6895	57.11	0.7132	56.03	0.7370	55.02	0.7608	54.08	0.7846	53.19	0.8084	52.35	0.8321	51.55	0.8559	50.80	0.8797	50.09	0.9035	49.42	0.9272	48.77	0.9510	48.16	0.9748
57.87	0.6782	56.72	0.7016	55.64	0.7250	54.64	0.7484	53.69	0.7718	52.80	0.7952	51.96	0.8195	51.16	0.8419	50.41	0.8653	49.70	0.8887	49.03	0.9121	48.39	0.9355	47.78	0.9589
57.74	0.6743	56.59	0.6977	55.52	0.7210	54.51	0.7443	53.56	0.7675	52.67	0.7908	51.83	0.8140	51.04	0.8373	50.28	0.8605	49.57	0.8838	48.90	0.9071	48.26	0.9303	47.65	0.9536
57.68	0.6726	56.53	0.6958	55.45	0.7190	54.44	0.7422	53.50	0.7654	52.61	0.7886	51.76	0.8118	50.97	0.8350	50.22	0.8582	49.51	0.8814	48.83	0.9046	48.19	0.9278	47.58	0.9509
57.63	0.6714	56.48	0.6945	55.41	0.7176	54.40	0.7408	53.45	0.7639	52.56	0.7871	51.72	0.8102	50.93	0.8334	50.18	0.8565	49.47	0.8797	48.79	0.9028	48.15	0.9260	47.54	0.9491
57.50	0.6674	56.35	0.6904	55.27	0.7135	54.26	0.7365	53.32	0.7595	52.43	0.7825	51.59	0.8055	50.79	0.8285	50.04	0.8516	49.33	0.8746	48.66	0.8976	48.01	0.9206	47.40	0.9436
57.48	0.6670	56.33	0.6900	55.26	0.7130	54.25	0.7360	53.30	0.7590	52.41	0.7820	51.57	0.8050	50.78	0.8280	50.03	0.8510	49.32	0.8740	48.64	0.8970	48.00	0.9200	47.39	0.9430

n = 0.046		n = 0.047		n = 0.048		n = 0.049		n = 0.050	
y	x	y	x	y	x	y	x	y	x
48.61	1.2362	48.15	1.2631						
47.84	1.2006	47.38	1.2267	46.93	1.2528				
47.32	1.1768	46.86	1.2024	46.42	1.2280	45.99	1.2536		
46.95	1.1598	46.49	1.1851	46.05	1.2103	45.62	1.2355		
46.68	1.1471	46.21	1.1721	45.77	1.1970	45.34	1.2219	44.94	1.2468
46.46	1.1372	46.00	1.1620	45.56	1.1866	45.13	1.2114	44.72	1.2361
46.29	1.1293	45.83	1.1539	45.38	1.1784	44.96	1.2029	44.55	1.2275
46.51	1.0936	45.05	1.1174	44.61	1.1412	44.18	1.1650	43.77	1.1887
45.13	1.0758	44.66	1.0992	44.22	1.1226	43.79	1.1460	43.39	1.1693
45.00	1.0699	44.53	1.0931	44.09	1.1164	43.67	1.1395	43.26	1.1629
44.93	1.0669	44.47	1.0909	44.03	1.1133	43.60	1.1365	43.19	1.1597
44.89	1.0649	44.43	1.0881	43.98	1.1112	43.56	1.1343	43.15	1.1575
44.75	1.0587	44.29	1.0817	43.85	1.1047	43.42	1.1277	43.01	1.1507
44.74	1.0580	44.28	1.0811	43.83	1.1040	43.41	1.1270	43.00	1.1500

Formulas.

$$y = 23 + \frac{0.00155}{S} + \frac{1}{n} .$$

$$x = ny - 1 .$$